The problem ran approximately 15 sec on a CDC 6600 computer (including compilation time).

Solution for the Unconstrained Minimization Problem

The solution of the problem without size constraints (only necessary that the masses be nonnegative) is summarized in Fig. 4. Convergence to a numerical solution was effected after nine cycles, with a structural weight reduction of approximately 48%. However, because of the unrealistic absence of minimum-size constraints, the spar caps are driven down in weight to the point where they are almost entirely eliminated, whereas the skin gages stabilize at values which prevent wing divergence.

This design would be inadequate from almost any strength or stiffness standpoint, since the skins alone provide little in the way of bending material.

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Minimum Time and Minimum Fuel Flight Path Sensitivity

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The sensitivity of minimum time and minimum fuel flight paths to variations in aircraft parameters in different atmospheric conditions was investigated using the energy state approximation. Numerical results are presented for a typical supersonic aircraft in Standard Day, Hot Day, and Cold Day atmospheres. Minimum time and minimum fuel flight paths in the vertical plane are well documented in the literature. This paper shows how flight time and fuel consumption are affected by changes in thrust, weight, drag coefficients (C_{Do} and C_{Di}), and specific fuel consumption (SFC) in each of three different atmospheric conditions (Standard Day, Hot Day, and Cold Day). For each variation, the effect on performance (flight time or fuel consumption) is determined for the nominal paths. Then for each variation, the flight path is adjusted to be either time optimal or fuel optimal. As a result of this analysis, it was found that flight time and fuel consumption are sensitive to variations in C_{Do}, SFC, and aircraft weight. There is only a slight sensitivity to variations in C_{Di} and no flight time sensitivity to variations in SFC. It was found that a Hot Day atmosphere tended to degrade aircraft performance by increasing flight time and fuel consumption along the flight path, while a Cold Day atmosphere tended to enhance performance. Adjusting the nominal flight paths to be time optimal or fuel optimal for the conditions being considered was found to be desirable for only a limited number of conditions. In a Standard Day atmosphere, the only conditions for which path adjustment significantly improved performance are large thrust reductions and large increases in C_{Do}. In a Hot Day atmosphere, path adjustment improved performance for every variation with the exception of thrust increases and C_{Do} decreases. In a Cold Day atmosphere, path adjustment failed to significantly improve performance for any variation.

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Nomenclature

C_{Do} = drag coefficient at zero lift
C_{Di} = induced drag coefficient
D = drag, lb
E = specific aircraft energy, ft
g = gravitational acceleration, ft/sec^2

h = altitude, ft
T = thrust, lb
σ = specific fuel consumption (SFC), hr^{-1}
V = velocity, fps
W = aircraft weight, lb
W_f = fuel weight consumed, lb

I. Introduction

FLIGHT path optimization for supersonic aircraft has been the topic of numerous studies during the last twenty years.[1-9] Most of these studies have been directed toward performance optimization assuming nominal aircraft and atmospheric conditions. Several reports [4,7,9] have investigated the sensitivity of these flight paths to parametric variations. This paper differs from these reports in that these parametric variations are considered not only in a Standard Day atmosphere [10] but also in a Hot Day atmosphere and a Cold Day atmosphere.

The purpose of this study is two-fold: 1) to use the energy-state approximation to analyze the sensitivity of nominal minimum time and minimum fuel paths to parametric and atmospheric variations, and 2) to determine the desirability of adjusting the flight paths to be either time optimal or fuel optimal for the variations under consideration. As we shall see, flight time and fuel consumption are sensitive to parametric variations and this sensitivity is increased in a Hot Day atmosphere and reduced in a Cold Day atmosphere.

The energy state approximation has proven successful in earlier investigations and greatly simplifies computations. In this approximation, the total specific energy of the aircraft, $E = h + 1/2\ V^2/g$, becomes a state variable with velocity as a control variable and altitude as an auxiliary variable. This relationship gives rise to the possibility of an instantaneous exchange of aircraft kinetic energy with potential energy while the total aircraft energy remains constant. This instantaneous energy exchange is approximated by "zoom" dives or climbs.[1] In order to minimize the flight time between any two energy levels, the specific excess power, $\dot{E} = V(T - D)/W$, must be maximized at each point along the flight path. Similarly, in order to minimize the fuel consumption, the specific energy per pound of fuel consumed, $dE/dW = V(T - D)/W(\sigma T)$, must be maximized at each point along the flight path.[1]

The parameters used in this sensitivity study are thrust, the coefficients of drag (C_{Do} and C_{Di}), aircraft weight, and specific fuel consumption (SFC). Flight time and fuel consumption sensitivity to variations in these parameters are analyzed in a Standard Day, Hot Day, and a Cold Day atmosphere. The atmospheric parameters (temperature, air density, and temperature profiles) for the Standard Day atmosphere are specified by the U.S. Standard Atmosphere (1962),[10] while the parameters for a Hot Day and Cold Day atmosphere are specified by MIL-STD-210A.[11] The temperature and pressure profiles for each of these atmospheres are presented in Figs. 1 and 2. The nominal values for thrust, drag coefficients, weight, and SFC in a Standard Day atmosphere are used to determine the nominal minimum time and minimum fuel flight paths. After these nominal flight paths have been determined, flight time and fuel consumption sensitivity will be determined for parametric variations in thrust, drag coefficients, aircraft weight, and SFC in each of the three different atmospheres. Performance sensitivity will be defined as the percent change in flight time or fuel consumption in a Standard Day atmosphere along the respective nominal minimum time or minimum fuel paths. The second part of this analysis will determine flight paths adjusted to be time optimal and fuel optimal for parametric variations. By comparing the resulting improvement in performance of the adjusted path over the nominal path, we can determine the desirability of using the adjusted path.

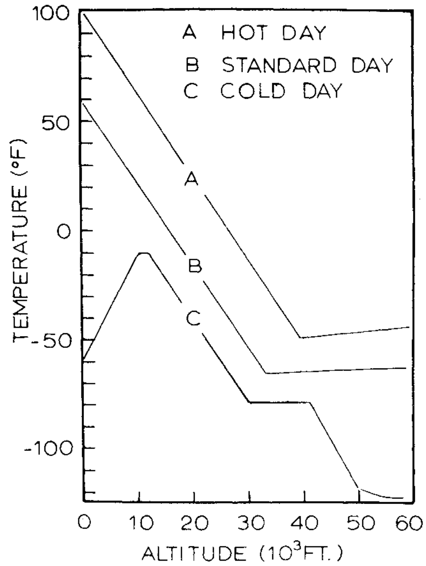

Fig. 1 Atmospheric temperature profile.

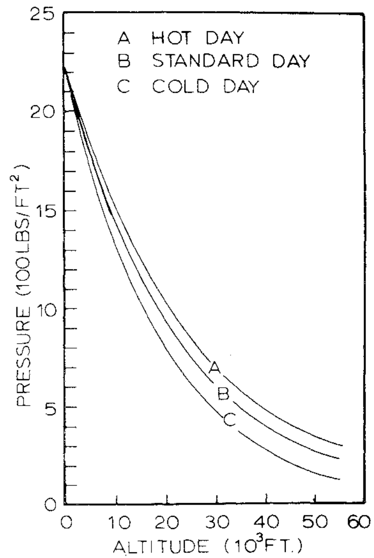

Fig. 2 Atmospheric pressure profile.

Two computer programs were developed to implement the analysis. The first program performed a search along the flight path for maximum values of excess power (minimum time path) and for maximum values of specific energy per pound of fuel consumed (minimum fuel path). Once the appropriate optimal path is determined, the flight time and fuel consumption for each parametric variation was determined in each of the three atmospheres using the second program. The resulting nominal minimum time and minimum fuel paths are shown in Figs. 3 and 4. Each flight path in this study starts at initial specific energy level E_o = 1000 ft and ends at final specific energy level E_f = 81,000 ft. Each optimal path exhibits the characteristic Rutowski path[5] with a zoom dive through Mach 1. This zoom dive is the result of the energy state assumption that kinetic energy and potential energy can be instantaneously interchanged. This assumption greatly simplifies computations, but it also introduces a problem; obviously the zoom dive does not physically occur in zero time nor with zero fuel consumption. However, this limitation in the energy state approximation has been shown by other investigations[1] to be a relatively minor disadvantage when compared to the many

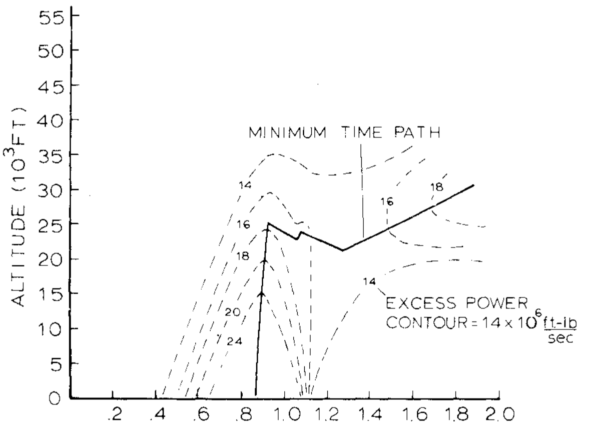

Fig. 3 Excess power contours and minimum time path for nominal conditions.

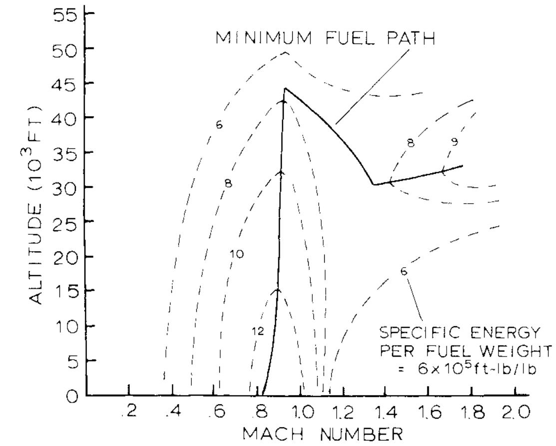

Fig. 4 Specific energy per fuel weight consumed contours and minimum fuel path for nominal conditions.

advantages gained by its use. If absolute numerical comparisons are required, the energy state approximation can be modified[12] to include approximations for the flight time and fuel consumed during the zoom dives. For example, if one computes the average velocity during the zoom and assumes a reasonable flight path angle during the zoom ($\approx 30°$) then the time and fuel consumed is approximated by $\Delta t = \Delta h / V_{AV} \sin(\gamma_{AV})$ and $\Delta W_f = (\sigma_{AV} \ T_{AV}) \ \Delta h / V_{AV} \sin(\gamma_{AV})$ where the subscript AV denotes average values during the zoom dive.

II. Numerical Results

A. Sensitivity Analysis‡

The flight time between any two specific energy levels is given by

$$t = \int_{E_1}^{E_2} [W/V(T-D)] dE$$

Thus flight time is inversely proportional to thrust and directly proportional to both drag terms and aircraft weight. Furthermore, variations in SFC has no effect on flight time. The fuel consumption between any two specific levels is given by

$$W_f = \int_{E_1}^{E_2} [\sigma W/V(1-D/T)] dE$$

Thus fuel consumption is inversely proportional to thrust and directly proportional to drag terms, aircraft weight, and SFC. Using a variation range of −20% to +20% for each of these parameters, the performance sensitivity for each parametric variation is presented in Tables 1–3 for a Standard Day, Hot Day, and Cold Day atmosphere, respectively.

‡Nominal aircraft parameters are shown in Table A-1 (from Ref.[1]) of the Appendix.

Table 1 Flight time and fuel consumption sensitivity to parametric variations along the nominal paths in a Standard Day atmosphere

Parameter	Variation (%)	Flight time (sec)	Sensitivity (%)	Fuel consumed (lb)	Sensitivity (%)
Nominal	0	160.1	0	2989.9	0
Thrust	20	110.3	−31.1	2684.8	−10.2
	10	130.1	−18.8	2809.8	−6.0
	−10	213.5	33.4	3281.9	9.8
	−20	350.0	118.6	3851.3	28.8
C_{Do}	20	233.1	45.6	3523.0	17.8
	10	186.9	16.7	3214.9	7.5
	−10	142.3	−11.1	2817.7	−5.8
	−20	129.4	−19.2	2677.8	−10.4
C_{Di}	20	161.8	1.1	3048.0	1.9
	10	160.9	0.5	3019.0	1.0
	−10	159.3	−0.5	2963.7	−0.9
	−20	158.5	−1.0	2937.2	−1.8
Weight	20	196.7	22.8	3745.8	25.3
	10	178.1	11.2	3356.0	12.2
	−10	142.7	−10.9	2645.8	−11.5
	−20	125.7	−21.5	2317.1	−22.5
SFC	20	160.1	0	3589.1	20.0
	10	160.1	0	3290.0	10.0
	−10	160.1	0	2691.9	−10.0
	−20	160.1	0	2392.8	−20.0

Table 2 Flight time and fuel consumption sensitivity to parametric variations along the nominal paths in a Hot Day atmosphere

Parameter	Variation (%)	Flight time (sec)	Sensitivity (%)	Fuel consumed (lb)	Sensitivity (%)
Nominal	0	265.9	66.1	3491.9	16.8
Thrust	20	150.2	−6.2	2959.2	−1.0
	10	189.0	18.1	3162.4	5.8
	−10	543.9	239.8	4150.4	38.8
	−20	a	a	6449.9	115.7
C_{Do}	20	2081.4	1200.1	4971.8	66.3
	10	434.3	171.3	3999.8	33.8
	−10	205.4	28.3	3169.0	6.0
	−20	173.0	8.1	2939.6	−1.7
C_{Di}	20	271.7	69.7	3582.2	19.8
	10	268.8	67.9	3536.2	18.3
	−10	263.2	64.4	3449.2	15.4
	−20	260.5	62.7	3407.8	14.0
Weight	20	334.8	109.1	4440.2	48.5
	10	299.2	86.9	3945.6	32.0
	−10	234.6	46.6	3070.8	2.7
	−20	205.0	28.1	2675.8	−10.5
SFC	20	265.9	66.1	4190.3	40.2
	10	265.9	66.1	3841.1	28.5
	−10	265.9	66.1	3142.7	5.1
	−20	265.9	66.1	2793.5	−6.6

[a] Aircraft could not follow nominal minimum time path (Fig. 3) in a Hot Day atmosphere.

Table 3 Flight time and fuel consumption sensitivity to parametric variations along the nominal paths in a Cold Day atmosphere

Parameter	Variation (%)	Flight time (sec)	Sensitivity (%)	Fuel consumed (lb)	Sensitivity (%)
Nominal	0	129.8	−18.9	2786.4	−6.8
Thrust	20	94.4	−41.0	2557.5	−14.5
	10	109.1	−31.9	2653.4	−11.3
	−10	162.1	1.3	2985.4	−.1
	−20	222.2	38.3	3323.6	11.2
C_{Do}	20	162.4	1.4	3126.4	4.6
	10	143.4	−10.4	2937.5	−1.8
	−10	119.5	−25.3	2661.4	−11.0
	−20	111.4	−30.4	2555.2	−14.5
C_{Di}	20	130.8	−18.3	2827.7	−5.3
	10	130.3	−18.6	2809.2	−6.0
	−10	129.3	−19.2	2764.1	−7.5
	−20	128.8	−19.5	2742.5	−8.3
Weight	20	158.5	−1.0	3470.9	16.1
	10	144.0	−10.1	3118.7	4.3
	−10	116.0	−27.6	2470.2	−17.4
	−20	102.4	−36.0	2167.3	−27.5
SFC	20	129.8	−18.9	3343.6	11.8
	10	129.8	−18.9	3065.0	2.5
	−10	129.8	−18.9	2507.7	−16.1
	−20	129.8	−18.9	2229.1	−25.4

Table 4 Flight time and fuel consumption sensitivity to parametric variations along the adjusted paths in a Standard Day atmosphere

Parameter	Variation (%)	Flight time (sec)	Sensitivity (%)	Fuel consumed (lb)	Sensitivity (%)
Nominal	0	160.0	0	2989.0	0
Thrust	20	107.6	−32.8	2669.5	−10.7
	10	129.0	−19.4	2801.4	−6.3
	−10	208.6	30.3	3269.7	9.4
	−20	297.9	86.1	3793.2	26.9
C_{Do}	20	213.3	33.3	3470.9	16.1
	10	183.3	14.5	3200.8	7.1
	−10	140.9	−12.0	2805.4	−6.2
	−20	124.8	−22.0	2648.0	−11.4
C_{Di}	20	161.8	1.1	3039.8	1.7
	10	160.9	0.5	3012.7	.8
	−10	159.3	−0.5	2958.3	−1.1
	−20	158.4	−1.0	2931.2	−2.0
Weight	20	196.6	22.8	3726.0	24.6
	10	178.1	11.2	3348.8	12.0
	−10	142.7	−10.9	2640.5	−11.7
	−20	125.7	−21.5	2309.6	−22.8
SFC	20	160.1	0	3487.7	20.0
	10	160.1	0	3288.8	10.0
	−10	160.1	0	2690.8	−10.0
	−20	160.1	0	2391.8	−20.0

In a Standard Day atmosphere (Table 1), reducing thrust by 20% increases flight time by nearly 120%. Thus flight time is inversely proportional to thrust. In a Hot Day atmosphere (Table 2), reducing thrust by 10% increases flight time by nearly 240% and the aircraft fails to follow the nominal minimum time path (Fig. 3) when the thrust is reduced by 20%. Similar performance degradation is seen for each parametric variation in a Hot Day atmosphere. However, in a Cold Day atmosphere (Table 3), reducing thrust by 20% increases flight time by nearly 39%. Using the same analysis for each parametric variation, it is seen that flight time and fuel consumption are inversely proportional to thrust and directly proportional to drag terms, aircraft weight, and SFC (fuel consumption only). Furthermore, a Hot Day atmosphere reduces performance and a Cold Day improved performance. These results are demonstrated in Fig. 5 and Fig. 6 for thrust variations in each of the three atmospheres.

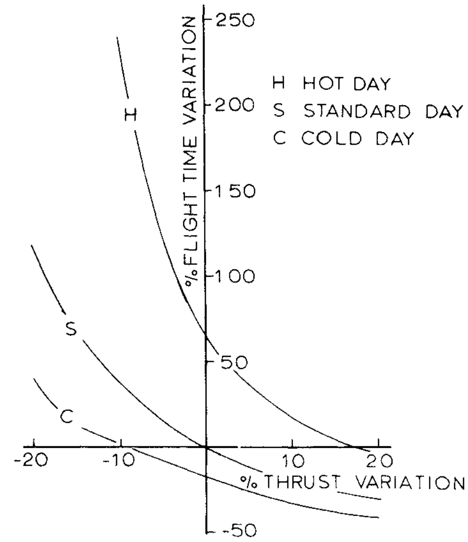

Fig. 5 Flight time sensitivity to thrust variations in three different atmospheres.

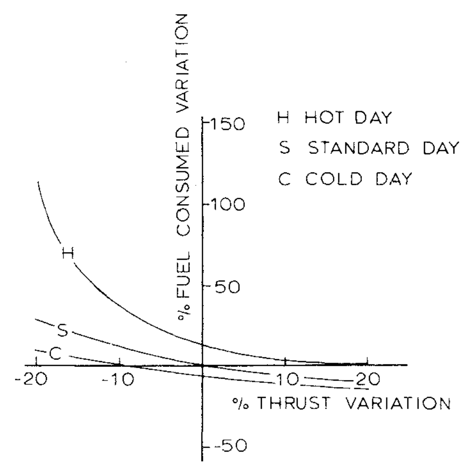

Fig. 6 Fuel consumption sensitivity to thrust variations in three different atmospheres.

B. Adjusted Optimal Flight Paths

Instead of using the nominal paths, the flight paths are adjusted to be time optimal or fuel optimal for each of the variations considered. The flight time and fuel consumption for each variation along these adjusted paths are shown in Table 4 (Standard Day), Table 5 (Hot Day), and Table 6 (Cold Day). Using thrust variations, it is seen that in a Standard Day the flight time and fuel consumption for adjusted paths (Table 4) shows significant improvement over the nominal path (Table 1) only for a 20% reduction in thrust and a 20% increase in C_{Do}. As for the other parametric variations in a Standard Day atmosphere, there is no significant improvement in using the adjusted paths. The adjusted paths in a Hot Day atmosphere (Table 5) show significant improvement for flight time performance along the nominal path in a Hot Day (Table 2) for each parametric variation. However, with the exception of a 20% thrust reduction and a 20% increase in C_{Do}, the adjusted path does not improve fuel consumption in a Hot Day. The adjusted paths in a Cold

Table 5 Flight time and fuel consumption sensitivity to parametric variations along the adjusted paths in a Hot Day atmosphere

Parameter	Variation (%)	Flight time (sec)	Sensitivity (%)	Fuel consumed (lb)	Sensitivity (%)
Nominal	0	231.2	44.4	3439.4	15.0
Thrust	20	149.7	−6.5	2979.5	−.3
	10	182.0	13.7	3163.4	5.8
	−10	317.2	98.2	3931.9	31.5
	−20	521.5	225.7	5187.2	73.5
C_{Do}	20	338.7	111.6	4313.7	44.3
	10	274.8	71.7	3789.8	26.8
	−10	199.1	24.3	3176.1	6.2
	−20	173.6	8.4	2964.7	−0.8
C_{Di}	20	236.2	47.5	3546.0	18.6
	10	233.7	46.0	3495.4	16.9
	−10	228.9	43.0	3389.0	13.4
	−20	226.7	41.6	3337.6	11.6
Weight	20	290.6	81.5	4410.4	47.5
	10	260.0	62.4	3906.5	30.7
	−10	204.2	27.5	3008.4	0.6
	−20	178.5	11.5	2602.8	−12.9
SFC	20	231.2	44.4	4127.3	38.0
	10	231.2	44.4	3783.3	26.5
	−10	231.2	44.4	3095.5	3.5
	−20	231.2	44.4	2751.5	−8.0

Table 6 Flight time and fuel consumption sensitivity to parametric variations along the adjusted paths in a Cold Day atmosphere

Parameter	Variation (%)	Flight time (sec)	Sensitivity (%)	Fuel consumed (lb)	Sensitivity (%)
Nominal	0	123.0	−23.2	2680.9	−10.3
Thrust	20	84.4	−47.3	2429.2	−18.7
	10	98.9	−38.2	2541.1	−15.0
	−10	156.7	−2.1	2885.2	−3.5
	−20	214.9	34.2	3225.5	7.9
C_{Do}	20	158.3	−1.1	3032.5	1.4
	10	139.6	−12.8	2841.4	−5.0
	−10	110.1	−31.2	2538.5	−15.1
	−20	98.1	−38.7	2404.2	−19.6
C_{Di}	20	124.6	−22.2	2716.2	−9.2
	10	123.3	−23.0	2696.4	−9.8
	−10	122.8	−23.3	2665.3	−10.8
	−20	121.4	−24.2	2647.5	−11.4
Weight	20	150.3	−6.1	3306.2	10.6
	10	136.1	−15.0	2989.5	−0.1
	−10	110.1	−31.2	2384.2	−20.2
	−20	98.1	−38.7	2098.0	−29.8
SFC	20	123.0	−23.2	3217.1	7.6
	10	123.0	−23.2	2949.0	−1.4
	−10	123.0	−23.2	2412.8	−19.3
	−20	123.0	−23.2	2144.7	−28.3

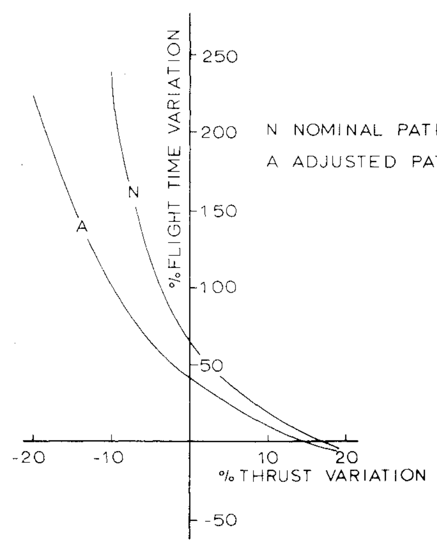

Fig. 7 Flight time sensitivity to thrust variations for a nominal path and an adjusted path in a Hot Day atmosphere.

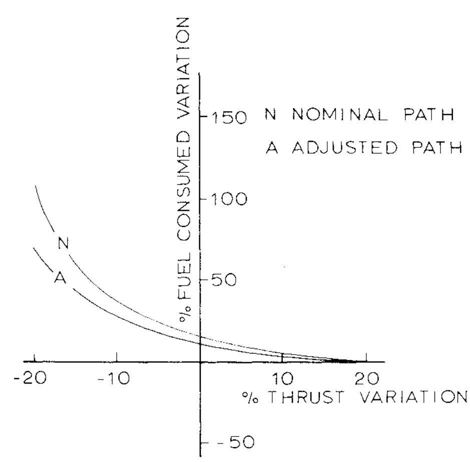

Fig 8 Fuel consumption sensitivity to thrust variations for a nominal path and an adjusted path in a Hot Day atmosphere.

Day atmosphere (Table 6) fail to significantly improve either flight time or fuel consumption over the nominal values (Table 3). A comparison of the flight time and fuel consumption of the adjusted flight paths with the flight time and fuel consumption of the nominal path is shown in Figs. 7 and 8 for thrust variations.

III. Conclusions

Flight time and fuel consumption along the nominal flight paths were found to be inversely proportional to thrust variations and directly proportional to variations in aircraft weight, drag coefficients, and specific fuel consumption (fuel consumption only). Flight time was more sensitive than fuel consumption to parametric variations. As expected, a Hot Day atmosphere tended to degrade aircraft performance while a Cold Day atmosphere improved performance. For example, when thrust was reduced by 10% in a Standard Day atmosphere, flight time increased by nearly 120% and fuel consumption by nearly 10%. In a Hot Day atmosphere, flight time increased by 240% and fuel consumption by 39%, while in a Cold Day atmosphere, flight time increased by only 1% and fuel consumption was actually slightly below its nominal value. Adjusting the flight path to be either time optimal or fuel optimal was effective for only a limited number of cases. Adjusting the flight path to be time optimal for either a 20% thrust reduction or a 20% C_{Do} increase in a Standard Day atmosphere improved flight time between initial and final energy levels. However, with path adjustment, there is no significant flight time improvement over the nominal path flight time for any other parametric variation, and adjusting the path to be fuel optimal resulted in no significant fuel savings over the nominal fuel consumption for any parametric variation in a Standard Day atmosphere. In a Hot Day, with the exception of thrust increases and C_{Do} decreases, path adjustment improved flight time for each parametric variation. Path adjustment significantly improved fuel consumption in a

Table A-1 Lift and drag coefficients as a function of Mach number

Mach No.	$C_{L\alpha}$	C_{D_0}	η
0.0–0.8	3.44	0.013	0.54
0.9	3.58	0.014	0.75
1.0	4.44	0.031	0.79
1.2	3.44	0.041	0.85
1.4	3.01	0.039	0.89
1.6	2.86	0.036	0.93
1.8–2.0	2.44	0.035	0.93

$C_L = C_{L_\alpha}\alpha$ $\quad L = C_L {}^1\!/_2\rho V^2 S$

$C_D = C_{D_0} + \eta C_{L_\alpha}\alpha^2$ $\quad D = C_D {}^1\!/_2\rho V^2 S$

$S = 530$ ft².

Table A-2 Thrust as a function of Mach number and altitude for a Standard Day atmosphere

Mach	Thrust (10³ lb) Altitude (10³ ft)						
No.	0	5	15	25	35	45	55
0.0	35.1	30.1					
0.2	34.1	29.2	20.7				
0.4	35.0	30.4	21.8	14.9	9.7		
0.6	36.6	32.8	24.6	16.9	11.1	6.7	4.0
0.8	40.6	36.6	28.7	20.3	13.4	8.2	4.8
1.0	44.7	42.6	33.2	22.6	16.7	10.3	6.1
1.2		45.2	39.2	29.3	20.9	12.9	7.7
1.4			44.1	34.9	25.2	15.7	9.4
1.6				40.0	30.0	18.8	11.4
1.8				43.1	34.5	21.7	13.2

Table A-3 Thrust as a function of Mach number and altitude for a Hot Day atmosphere

Mach	Thrust (10³ lb) Altitude (10³ ft)						
No.	0	5	15	25	35	45	55
0.0	31.0	27.4					
0.2	29.1	26.4	19.9				
0.4	30.4	26.7	20.5	14.3	9.3		
0.6	32.0	28.2	21.9	16.1	10.6	6.4	3.8
0.8	34.5	31.2	24.6	18.8	12.8	7.8	4.6
1.0	37.2	35.6	28.9	21.8	15.7	9.8	5.8
1.2		38.4	33.1	26.0	18.5	11.9	7.0
1.4			36.2	29.6	22.1	14.2	8.4
1.6				32.4	25.5	16.6	9.8
1.8					27.7	18.6	11.0

Table A-4 Thrust as a function of Mach number and altitude for a Cold Day atmosphere

Mach	Thrust (10³ lb) Altitude (10³ ft)						
No.	0	5	15	25	35	45	55
0.0	39.4	31.6					
0.2	38.8	31.0	21.4				
0.4	41.2	32.8	22.7	15.4			
0.6	46.9	37.2	25.7	17.6	11.4		
0.8	52.7	44.4	30.8	21.2	13.8	8.6	
1.0	51.8	50.8	37.6	26.2	17.2	10.7	6.7
1.2		48.2	44.2	32.8	21.6	13.6	8.6
1.4			49.6	39.3	27.0	17.2	11.0
1.6				46.5	32.1	20.8	14.0
1.8				49.2	37.8	25.0	17.1

Hot Day for only a 20% thrust reduction and a 20% C_{Do} increase. In a Cold Day, path adjustment was ineffective in improving either flight time or fuel consumption for any parametric variation.

Appendix

Data for thrust, drag, and lift are given in Tables A1–A4. The nominal aircraft weight is 40,000 lb. The fuel used is assumed to be JP4 having a nominal density of 6.5 lb/gal at 65 °F. The specific fuel consumption has a typical range of values from 2.0 hr^{-1} to 2.5 hr^{-1}. In order to simplify computations an average value of 2.25 hr^{-1} was chosen as a nominal SFC.

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